Final state interactions: from strangeness to beauty*

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Abstract

I give a brief review of final state interactions in meson decays. I describe possible effects of FSI in K, D and B systems, paying particular attention to the description of the heavy meson decays. Available theoretical methods for dealing with the effects of FSI are discussed.

1   Motivation

Final state interactions (FSI) play an important role in meson decays. The presence of FSI forces one to consider several coupled channels, so their net effect might be significant, especially if one is interested in rare decays. This obvious observation, of course, does not exhaust the list of the motivations for better understanding of FSI. Many important observables that are sensitive to New Physics could also receive contributions from the final state rescatterings. An excellent example is provided by the T-violating lepton polarizations in K decays (such as $K \to \pi l \nu$ and $K \to \gamma l \nu$) that are not only sensitive to New Physics but could also be induced by the electromagnetic FSI. However, the most phenomenologically important effect of FSI is in the decays of B and D mesons used for studies of direct CP-violation, where one compares the rates of a B or D meson decay with the charged conjugated process [1]. The corresponding asymmetries, in order to be non-zero, require two different final states produced by different weak amplitudes which can go into each other by a strong interaction rescattering and therefore depend on both weak CKM phase and strong rescattering phase provided by the FSI. Thus, FSI directly affect the asymmetries and

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their size can be interpreted in terms of fundamental parameters *only* if these FSI phases are calculable. In all of these examples FSI complicates the interpretation of experimental observables in terms of fundamental parameters [2, 3]. In this talk I review the progress in understanding of FSI in meson decays.

The difference of the physical picture at the energy scales relevant to K, D and B decays calls for a specific descriptions for each class of decays. For instance, the relevant energy scale in K decays is $m_K \ll 1$ GeV. With such a low energy release only a few final state channels are available. This significantly simplifies the theoretical understanding of FSI in kaon decays. In addition, chiral symmetry can also be employed to assist the theoretical description of FSI in K decays. In D decays, the relevant scale is $m_D \sim 1$ GeV. This region is populated by the light quark resonances, so one might expect their significant influence on the decay rates and CP-violating asymmetries. No model-independent description of FSI is available, but it is hinted at experimentally that the number of available channels is still limited, allowing for a modeling of the relevant QCD dynamics. Finally, in B decays, where the relevant energy scale $m_B \gg 1$ GeV is well above the resonance region, the heavy quark limit might prove useful.

2   Some formal aspects of FSI

Final state interactions in $A \to f$ arise as a consequence of the unitarity of the $\mathcal{S}$-matrix, $\mathcal{S}^\dagger \mathcal{S} = 1$, and involve the rescattering of physical particles in the final state. The $\mathcal{T}$-matrix, $\mathcal{T} = i\,(1 - \mathcal{S})$, obeys the optical theorem:

$$\mathcal{D}isc\ \mathcal{T}_{A \to f} \equiv \frac{1}{2i}\left[\langle f|\mathcal{T}|A\rangle - \langle f|\mathcal{T}^\dagger|A\rangle\right] = \frac{1}{2}\sum_i \langle f|\mathcal{T}^\dagger|i\rangle\langle i|\mathcal{T}|A\rangle \ , \qquad (1)$$

where $\mathcal{D}isc$ denotes the discontinuity across physical cut. Using CPT in the form $\langle \bar f|\mathcal{T}|\bar A\rangle^* = \langle \bar A|\mathcal{T}^\dagger|\bar f\rangle = \langle f|\mathcal{T}^\dagger|A\rangle$, this can be transformed into

$$\langle \bar f|\mathcal{T}|\bar A\rangle^* = \sum_i \langle f|\mathcal{S}^\dagger|i\rangle\langle i|\mathcal{T}|A\rangle \ . \qquad (2)$$

Here, the states $|i\rangle$ represent all possible final states (including $|f\rangle$) which can be reached from the state $|A\rangle$ by the weak transition matrix $\mathcal{T}$. The right hand side of Eq. (2) can then be viewed as a weak decay of $|A\rangle$ into $|i\rangle$ followed by a strong rescattering of $|i\rangle$ into $|f\rangle$. Thus, we identify $\langle f|\mathcal{S}^\dagger|i\rangle$ as a FSI rescattering of particles. Notice that if $|i\rangle$ is an eigenstate of $\mathcal{S}$ with a phase $e^{2i\delta}$, we have

$$\langle \bar i|\mathcal{T}|\bar A\rangle^* = e^{-2i\delta_i}\langle i|\mathcal{T}|A\rangle \ . \qquad (3)$$

which implies equal rates for the charge conjugated decays[1]. Also

$$\langle \bar{i}|\mathcal{T}|\bar{B}\rangle = e^{i\delta}T_i\langle i|\mathcal{T}|A\rangle = e^{i\delta}T_i^* \tag{4}$$

The matrix elements T_i are assumed to be the "bare" decay amplitudes and have no rescattering phases. This implies that these transition matrix elements between charge conjugated states are just the complex conjugated ones of each other. Eq. (4) is known as Watson's theorem [4]. Note that the problem of unambiguous separation of "true bare" amplitudes from the "true FSI" ones (known as Omnés problem) was solved only for a limited number of cases.

2.1 K decays

The low scale associated with K decays suggests an effective theory approach of integrating out heavy particles and making use of chiral symmetry of QCD. This theory has been known for a number of years as chiral perturbation theory (χPT), which makes use of the fact that kaons and pions are the Goldstone bosons of chiral $SU(3)_L \times SU(3)_R$ broken down to $SU(3)_V$, and are the only relevant degrees of freedom at this energy. χPT allows for a consistent description of the strong and electromagnetic FSI in kaon system.

The discussion of strong FSI is naturally included in the χPT calculations of kaon decays processes at one or more loops [5]. In addition, kaon system is rather unique for its sensitivity to the electromagnetic final state interaction effects. Normally, one expects this class of corrections to be negligibly small. However, in some cases they are still very important. For instance, it is known that in non-leptonic K decays the $\Delta I = 1/2$ isospin amplitude is enhanced compared to the $\Delta I = 3/2$ amplitude by approximately a factor of 22. Since electromagnetism does not respect isospin symmetry, one might expect that electromagnetic FSI might contribute to the $\Delta I = 3/2$ amplitude at the level of $22/137 \sim 20\%$! Of course, some cancellations might actually lower the impact of this class of FSI [6].

There is a separate class of observables that is directly affected by electromagnetic FSI. It includes the T-violating transverse lepton polarizations in the decays $K \to \pi l \nu$ and $K \to l \nu \gamma$

$$P_l^\perp = \frac{\vec{s}_l \cdot (\vec{p}_i \times \vec{p}_l)}{|\vec{p}_i \times \vec{p}_l|}, \tag{5}$$

where $i = \gamma, \pi$. Observation of these polarizations in the currently running experiments implies an effect induced by New Physics.

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[1]This fact will be important in the studies of CP-violating asymmetries as no CP asymmetry is generated in this case.

A number of parameters of various extensions of the Standard Model can be constrained via these measurements [7]. It is, however, important to realize that the polarizations as high as $10^{-3}(10^{-6})$ could be generated by the electromagnetic rescattering of the final state lepton and pion or due to other intermediate states. These corrections have been estimated for a number of experimentally interesting final states [8].

2.2 D decays

The relatively low mass of the charm quark puts the D mesons in the region populated by the higher excitations of the light quark resonances. It is therefore natural to assume that the final state resacttering is dominated by the intermediate resonance states [9]. Unfortunately, no model-independent description exists at this point. Yet, the wealth of experimental results allows for the introduction of testable models of FSI [10]. These models are very important in the studies of direct CP-violating asymmetries

$$A_{CP} = \frac{\Gamma(D \to f) - \Gamma(\bar{D} \to \bar{f})}{\Gamma(D \to f) + \Gamma(\bar{D} \to \bar{f})} \sim \sin\theta_w \sin\delta_s, \qquad (6)$$

which explicitly depend on the values of both weak (θ_w) and strong (δ_s) phases. In most models of FSI in D decay, the phase δ_s is generated by the width of the nearby resonance and by calculating the imaginary part of loop integral with the final state particles coupled to the nearby resonance.

It is important to realize that the large final state interactions and the presence of the nearby resonances in the D system has an immediate impact on the $D - \bar{D}$ mixing parameters. It is well known that the short distance contribution to Δm_D and $\Delta\Gamma$ is very small, of the order of 10^{-18} GeV. Nearby resonances can enhance them by one or two orders of magnitude [11]. In addition, they provide a source for quark-hadron duality violations, as they populate the gap between the QCD scale and the scale set by the mass of the heavy quark normally required for the application of heavy quark expansions.

2.3 B decays

In the B system, where the density of the available resonances is large due to the increased energy, a different approach must be employed. One can use the fact that the $b-$quark mass is large compared to the QCD scale and investigate the behavior of final state phases in the $m_b \to \infty$ limit.

Significant energy release in B decays allows the studies of inclusive quantities, for instance inclusive CP-violating asymmetries of the form of Eq. (6). There, one can use duality arguments to calculate final state phases for charmless B decays using perturbative QCD [12]. Indeed, $b \to c\bar{c}s$

process, with subsequent final state rescattering of the two charmed quarks into the final state (penguin diagram) does the job, as for the energy release of the order $m_b > 2m_c$ available in b decay, the rescattered c-quarks can go on-shell generating a CP conserving phase and thus $\mathcal{A}_{CP}^{dir}$, which is usually quite small for the experimentally feasible decays, $\mathcal{O}(1\%)$. It is believed that larger asymmetries can be obtained in exclusive decays. However, a simple picture is lost because of the absence of the duality argument.

It is known that scattering of high energy particles may be divided into 'soft' and 'hard' parts. Soft scattering occurs primarily in the forward direction with limited transverse momentum distribution which falls exponentially with a scale of order 0.5 GeV. At higher transverse momentum one encounters the region of hard scattering, which can be described by perturbative QCD. In exclusive B decay one faces the difficulty of separating the two. It might prove useful to employ unitarity in trying to describe FSI in exclusive B decays.

It is easy to investigate first the *elastic* channel. The inelastic channels have to share a similar asymptotic behavior in the heavy quark limit due to the unitarity of the $\mathcal{S}$-matrix. The choice of elastic channel is convenient because of the optical theorem which connects the forward (imaginary) invariant amplitude $\mathcal{M}$ to the total cross section,

$$\mathcal{I}m\ \mathcal{M}_{f \to f}(s,\ t=0) = 2k\sqrt{s}\sigma_{f \to \text{all}} \sim s\sigma_{f \to \text{all}}\ \ , \tag{7}$$

where s and t are the usual Mandelstam variables. The asymptotic total cross sections are known experimentally to rise slowly with energy and can be parameterized by the form [13], $\sigma(s) = X\,(s/s_0)^{0.08} + Y\,(s/s_0)^{-0.56}$, where $s_0 = \mathcal{O}(1)$ GeV is a typical hadronic scale. Considering only the imaginary part of the amplitude, and building in the known exponential fall-off of the elastic cross section in t $(t < 0)$ [14] by writing

$$i\mathcal{I}m\ \mathcal{M}_{f \to f}(s,t) \simeq i\beta_0 \left(\frac{s}{s_0}\right)^{1.08} e^{bt}\ \ , \tag{8}$$

one can calculate its contribution to the unitarity relation for a final state $f = ab$ with kinematics $p'_a + p'_b = p_a + p_b$ and $s = (p_a + p_b)^2$:

$$\begin{aligned}
\mathcal{D}isc\ \mathcal{M}_{B \to f} &= \frac{-i}{8\pi^2} \int \frac{d^3 p'_a}{2E'_a} \frac{d^3 p'_b}{2E'_b} \delta^{(4)}(p_B - p'_a - p'_b) \mathcal{I}m\ \mathcal{M}_{f \to f}(s,t)\mathcal{M}_{B \to f} \\
&= -\frac{1}{16\pi} \frac{i\beta_0}{s_0 b} \left(\frac{m_B^2}{s_0}\right)^{0.08} \mathcal{M}_{B \to f}\ \ , \tag{9}
\end{aligned}$$

where $t = (p_a - p'_a)^2 \simeq -s(1 - \cos\theta)/2$, and $s = m_B^2$.

One can refine the argument further, since the phenomenology of high energy scattering is well accounted for by the Regge theory [14]. In the

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Regge model, scattering amplitudes are described by the exchanges of Regge trajectories (families of particles of differing spin) with the leading contribution given by the Pomeron exchange. Calculating the Pomeron contribution to the elastic final state rescattering in $B \to \pi\pi$ one finds [15]

$$\mathcal{D}isc \; \mathcal{M}_{B\to\pi\pi}|_{\text{Pomeron}} = -i\epsilon\mathcal{M}_{B\to\pi\pi}, \qquad \epsilon \simeq 0.21 \; . \qquad (10)$$

It is important that the Pomeron-exchange amplitude is seen to be almost purely imaginary. However, of chief significance is the identified weak dependence of ϵ on m_B – the $(m_B^2)^{0.08}$ factor in the numerator is attenuated by the $\ln(m_B^2/s_0)$ dependence in the effective value of b.

The analysis of the elastic channel suggests that, at high energies, FSI phases are *mainly generated by inelastic effects*, which follows from the fact that the high energy cross section is mostly inelastic. This also follows from the fact that the Pomeron elastic amplitude is almost purely imaginary. Since the study of elastic rescattering has yielded a $\mathcal{T}$-matrix element $\mathcal{T}_{ab\to ab} = 2i\epsilon$, i.e. $\mathcal{S}_{ab\to ab} = 1 - 2\epsilon$, and since the constraint of unitarity of the $\mathcal{S}$-matrix implies that the off-diagonal elements are $\mathcal{O}(\sqrt{\epsilon})$, with ϵ approximately $\mathcal{O}(m_B^0)$ in powers of m_B and numerically $\epsilon < 1$, then the inelastic amplitude must also be $\mathcal{O}(m_B^0)$ with $\sqrt{\epsilon} > \epsilon$. Similar conclusions follow from the consideration of the final state unitarity relations. This complements the old Bjorken picture of heavy meson decay (the dominance of the matrix element by the formation of the small hadronic configuration which grows into the final state pion "far away" from the point it was produced and does not interact with the soft gluon fields present in the decay, see also [16] for the discussion) by allowing for the rescattering of multiparticle states, production of whose is favorable in the $m_b \to \infty$ limit, into the two body final state. Analysis of the final-state unitarity relations in their general form is complicated due to the many contributing intermediate states, but we can illustrate the systematics of inelastic scattering in a two-channel model. It involves a two-body final state f_1 undergoing elastic scattering and a final state f_2 which represents 'everything else'. As before, the elastic amplitude is purely *imaginary*, which dictates the following *one-parameter* form for the S matrix

$$S = \begin{pmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{pmatrix} , \qquad T = \begin{pmatrix} 2i\sin^2\theta & \sin 2\theta \\ \sin 2\theta & 2i\sin^2\theta \end{pmatrix} , \quad (11)$$

where we identify $\sin^2\theta \equiv \epsilon$. The unitarity relations become

$$\mathcal{D}isc \; \mathcal{M}_{B\to f_1} = -i\sin^2\theta\mathcal{M}_{B\to f_1} + \frac{1}{2}\sin 2\theta\mathcal{M}_{B\to f_2} \; ,$$

$$\mathcal{D}isc \; \mathcal{M}_{B\to f_2} = \frac{1}{2}\sin 2\theta\mathcal{M}_{B\to f_1} - i\sin^2\theta\mathcal{M}_{B\to f_2} \qquad (12)$$

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Denoting $\mathcal{M}_1^0$ and $\mathcal{M}_2^0$ to be the decay amplitudes in the limit $\theta \to 0$, an exact solution to Eq. (12) is given by

$$\mathcal{M}_{B \to f_1} = \cos\theta \mathcal{M}_1^0 + i \sin\theta \mathcal{M}_2^0 , \qquad \mathcal{M}_{B \to f_2} = \cos\theta \mathcal{M}_2^0 + i \sin\theta \mathcal{M}_1^0 . \quad (13)$$

Thus, the phase is given by the inelastic scattering with a result of order

$$\mathcal{I}m \, \mathcal{M}_{B \to f} / \mathcal{R}e \, \mathcal{M}_{B \to f} \sim \sqrt{\epsilon} \left(\mathcal{M}_2^0 / \mathcal{M}_1^0 \right) . \quad (14)$$

Clearly, for physical B decay, we no longer have a simple one-parameter $\mathcal{S}$ matrix, and, with many channels, cancellations or enhancements are possible for the sum of many contributions. However, the main feature of the above result is expected to remain: inelastic channels cannot vanish and provide the FSI phase which is systematically of order $\sqrt{\epsilon}$ and thus does not vanish in the large m_B limit.

A contrasting point of view is taken in a recent calculation [17]. The argument is based on the *perturbative* factorization of currents (i.e. the absence of infrared singularities) in the matrix elements of four quark operators in the Bjorken setup. It is claimed that the leading contribution is given by the naive factorization result with non-leading corrections suppressed by α_s or $1/m_b$ (see, however, [18]). However, the role of multihadron intermediate states is not yet clear in this approach. Moreover, even accepting the result of [17], it would be premature to claim that the theory of exclusive B decays to light mesons is free of hadronic uncertainties. In fact, many important long distance final state rescattering effects involve exchange of global quantum numbers, such as charge or strangeness, and thus are suppressed by $\approx 1/m_B$. These were shown to be important and can be quite large [2, 19]. This is easy to see in the Regge description of FSI where this exchange is mediated by the ρ and/or higher lying trajectories. This fact raises a question whether the scale $m_b \approx 5$ GeV is large enough for the asymptotic limit to set.

(i) Bounds on the FSI Corrections. In view of the large theoretical uncertainties [20] involved in the calculation of the FSI contributions, it would be extremely useful to find a phenomenological method by which to bound the magnitude of the FSI contribution. The observation of a larger asymmetry would then be a signal for New Physics. Here the application of flavor $SU(3)$ flavor symmetry provides powerful methods to obtain a direct upper bound on the FSI contribution [19]. The simplest example involves bounding FSI in $B \to \pi K$ decays using $B^\pm \to K^\pm K$ transitions [2].

(ii) Direct Observation. Another interesting way of studying FSI involves rare weak decays for which the direct amplitude $A(B \to f)$ is suppressed compared to $A(B \to i)$. They offer a tantalizing possibility of the *direct observation* of the effects of FSI.

One of the possibilities involves dynamically suppressed decays which proceed via weak annihilation diagrams. It has been argued that final state

interactions, if large enough, can modify the decay amplitudes, violating the expected hierarchy of amplitudes. For instance, it was shown [21] that the rescattering from the dominant tree level amplitude leads to the suppression of the weak annihilation amplitude by only $\lambda \sim 0.2$ compared to $f_B/m_B \sim \lambda^2$ obtained from the naive quark diagram estimate.

Alternatively, one can study OZI-violating modes, i.e. the modes which cannot be realized in terms of quark diagrams without annihilation of at least one pair of the quarks, like $\overline{B}_d^0 \to \phi\phi, D^0\phi$ and $J/\psi\phi$. Unitarity implies that this decay can also proceed via the OZI-allowed weak transition followed by final state rescattering into the final state under consideration [22, 23]. In B-decays these OZI-allowed steps involve multiparticle intermediate states and might provide a source for violation of the OZI rule. For instance, the FSI contribution can proceed via $\overline{B}_d^0 \to \eta^{(\prime)}\eta^{(\prime)} \to \phi\phi$, $\overline{B}_d^0 \to D^{*0}\eta^{(\prime)} \to D^0\phi$ and $\overline{B}_d^0 \to \psi'\eta^{(\prime)} \to J/\psi\phi$. The intermediate state also includes additional pions. The weak decay into the intermediate state occurs at tree level, through the $(u\overline{u} + d\overline{d})/\sqrt{2}$ component of the $\eta^{(\prime)}$ wavefunction, whereas the strong scattering into the final state involves the $s\overline{s}$ component. Hence the possibility of using these decay modes as direct probes of the FSI contributions to B decay amplitudes. It is however possible to show that there exist strong cancellations [22] among various *two body* intermediate channels. In the example of $\overline{B}_d^0 \to \phi\phi$, the cancellation among η and η' is almost complete, so the effect is of the second order in the $SU(3)$-breaking corrections

$$Disc\ \mathcal{M}_{B\to\phi\phi} = O(\delta^2, \Delta^2, \delta\Delta) f_\eta F_0 A, \quad \delta = f_{\eta'} - f_\eta, \ \Delta = F_0' - F_0, \quad (15)$$

with $A \sim s^{\alpha_0-1} e^{i\pi\alpha_0/2}/8b$. This implies that the OZI-suppressed decays provide an excellent probe of the multiparticle FSI. Given the very clear signature, these decay modes could be probed at the upcoming B-factories.

In conclusion, I reviewed the physics of final state interactions in meson decays. One of the main goals of physics of CP violation and meson decay is to correctly extract the underlying parameters of the fundamental Lagrangian that are responsible for these phenomena. The understanding of final state interactions is very important for the success of this program.

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